Generalized Synthesis Methodology of Nonlinear Springs for Prescribed Load-Displacement Functions

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Nonlinear Spring Applications

- Statically Balanced Mechanisms
- MEMS Devices
- Design for crashworthiness
- Constant-force Springs
- Human Interfaces (Comfort, Tactile)
- Nature’s Nonlinear Compliance

Artificial Implants and Prosthetics
- Structures with synthesized nonlinear elasticities
- Mimic nonlinear and viscoelastic materials

Freedom Innovations
Otto Bock
Nonlinear Spring Parameterization

A typical nonlinear spring design

Network of splines
- For generating any nonlinear response

Input constrained along path
- Forces spring to stretch/compress (axial mode)

Curvilinear members
- Longer effective length
- Greater strain energy absorption
- Larger displacements and rotations
- Fewer stress concentrations

Pinned end conditions
- For large rotations and minimal bending stresses
Problem Statement

- Problem Statement

- Scope
  - Planar springs
  - Elastic range
  - No buckling
New Design Parameterization

- Topology

9 Splines
(3 Primary, 6 Secondary)
New Design Parameterization

• Each design has 96 variables that describe...

  **Topology**
  - Number of splines
  - Connection of splines
  - Boundary conditions

  **Shape**
  - Shape of the splines

  **Size**
  - In-plane thickness of the splines
# Design Examples

## 3 Nonlinear Springs (+ 1 Linear Spring)

<table>
<thead>
<tr>
<th>Shape-function</th>
<th>J-curve</th>
<th>S-curve</th>
<th>Constant-force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load-range</td>
<td>10N</td>
<td>75N</td>
<td>150N</td>
</tr>
<tr>
<td>Displacement-range</td>
<td>20mm</td>
<td>80mm</td>
<td>150mm</td>
</tr>
<tr>
<td>$N_{up}$ (Scaling)</td>
<td>1.2</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Square design space size (L)</td>
<td>100mm (10cm)</td>
<td>500mm (0.5m)</td>
<td>1000mm (1m)</td>
</tr>
<tr>
<td>Material modulus (material)</td>
<td>115MPa (Titanium)</td>
<td>115MPa (Titanium)</td>
<td>70MPa (Aluminum)</td>
</tr>
<tr>
<td>Maximum stress (safety factor)</td>
<td>830MPa (1)</td>
<td>415MPa (2)</td>
<td>275MPa (1)</td>
</tr>
<tr>
<td>Out-of-plane thickness</td>
<td>4mm</td>
<td>20mm</td>
<td>60mm</td>
</tr>
<tr>
<td>In-plane thickness</td>
<td>0.4-0.7mm</td>
<td>1-3mm</td>
<td>2-5mm</td>
</tr>
</tbody>
</table>

### Design Space

- **Width (L)**: 100mm (10cm) to 1000mm (1m)
- **Height (L)**: 100mm (10cm) to 1000mm (1m)
- **Material options**:
  - Titanium (115MPa)
  - Aluminum (70MPa)

### Genetic Algorithm
- Population: 96
- Crossover rate: 70%
- Mutation rate: 3%
Nonlinear Spring Applications

- Automotive seat cushion (hardening spring)
- Constant-force applications (softening spring)
Automotive Seat Cushion

4-inch foam cushion (Expensive to store and ship)

Rigid Seat Pan
*Multi-piece stamped and welded steel pan*
(No foam or cover shown)

EQUIVALENT LOAD-DISPLACEMENT FUNCTIONS

Current Design

- 4 Inches of Foam +
- Rigid Seat Pan

New Design

- 2 Inches of Foam +
- Compliant Seat Pan
  - (Nonlinear Mechanism/Spring)
Ford’s force-displacement data measured at the center of the seat cushion. [1]

4-inch foam cushion
Design a nonlinear spring:
1) Match load-displacement function
2) Fit within prescribed design space
Functional Description

Final Spring Design

- In-plane thickness = 0.027in (0.69mm)
- Out-of-plane thickness = 12in (304.8mm)
- Material = MartInsite M130 (E = 200 GPa)
- Max stress = 605MPa (< yield 930 MPa)
- Safety factor = 1.5
- Disp. = 29% of largest footprint dimension
Functional Description

Final Spring Design’s Assembly in Prototype

Remaining 2-inch foam cushion rests on plate

Nonlinear spring

2 springs in parallel

(Back) (Front)
Functional Design

Validation

Instron 8516

Nonlinear spring assembly (No foam included)
Functional Design

Validation (Instron 8516)

Nonlinear spring assembly (Foam included)
Conclusions

• Specifications were met only using 2-inches of foam
• Prescribed load-displacement function is sensitive to buckling
  – Original FEA design slightly buckled
  – Prototype did not buckle
  • Rotation of spring 2° accounts for discrepancy

- No buckling
- Shorter displacement-range
Results

FEA Prediction

Physical Deformation
Results and Conclusions

Validated:
- Deformation
- Shape function
- Displacement-range

Inconsistency:
- Load-range (15%)

In-plane thickness and load have cubed relationship

Decrease in-plane thickness by 0.06mm